## Invitation to the Doctoral Seminar

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"An extension of minimization based formulations and the projected gradient method with some applications"

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https://classroom.aatydakesday, 16 December 2020 © 10:00 a.m. 3nh-a2g

## Abstract

An inverse problem of reconstructing $x$ such that $A(x, u)=0$ from the data $y$ satisfying $C(u)=y$ can be written in the minimization form

$$
\operatorname{argmin}_{(x, u)}\left\{\mathscr{J}(x, u ; y):(x, u) \in M_{\mathrm{ad}}(y)\right\} .
$$

The classical approaches usually consider $\mathscr{J}(x, u ; y)=\frac{1}{2}\|C(x, u)-y\|^{2}$, $M_{\mathrm{ad}}=\{(x, u): A(x, u)=0\}$ or $\mathscr{J}(x, u ; y)=\frac{1}{2}\|A(x, u)\|^{2}, M_{\mathrm{ad}}=\{(x, u):$ $C(x, u)=y\}$. Here we follow a new approach assuming that the observation operator $C$ can be inverted on its range, as is the case, e.g., in the practically relevant setting of a finite dimensional observation space. In this case, we can split $u$ into two parts: the observed data part $\tilde{u}$ and the homogeneous data part $\hat{u}, u=\tilde{u}+\hat{u}$ where $C(\hat{u})=0$ and $C(\tilde{u})=y$. Let $C^{\text {ri }}$ be an right inverse operator of $C$, then the problem becomes to find $x$ such that

$$
A\left(x, C^{\mathrm{ri}}(y)+\hat{u}\right)=0, \quad \hat{u} \in \operatorname{Ker}(C):=\{\hat{u}: C(\hat{u})=0\}
$$

with the new minimization form which is to find

$$
(x, \hat{u}) \in \operatorname{argmin}_{(x, \hat{u})}\left\{\mathscr{\mathcal { L }}(x, \hat{u} ; y):(x, \hat{u}) \in M_{\mathrm{ad}}(y) .\right.
$$

Under some more conditions on the smoothness of $\mathscr{J}$ with respect to $y$, we can prove the well-definedness, stability and convergence of minimizers.
In practice, often an iterative regularization method will be applied to reconstruct $x$. Here we want to mention the projected gradient method

$$
x_{k+1}=\operatorname{Proj}_{M_{\mathrm{ad}}}\left(x_{k}-\mu_{k} \nabla J\left(x_{k}\right)\right)
$$

where $\operatorname{Proj}_{M_{\mathrm{ad}}}$ is the projection onto $M_{\mathrm{ad}}$ and $x$ stands for $(x, \hat{u})$, for brevity. This method can be applied to many problems. Here we show numerical results with a Matlab implementation for three examples: inverse groundwater filtration (GWF), impedance acoustic tomography (IAT), and electrical impedance tomography (EIT).

Barbara Kaltenbacher and the Department of Mathematics look forward to seeing you at the talk!

