ALPEN-ADRIA UNIVERSITÄT KLAGENFURT

INVITATION TO THE DOCTORAL SEMINAR

An extension of minimization based formulations and the projected gradient method with some applications

Kha Van Huynh | Universität Klagenfurt

Chaired by Barbara Kaltenbacher | Department of Mathematics

Wednesday, 16 December 2020 | 10:00–10:45

https://classroom.aau.at/b/bka-3nh-a2g

ABSTRACT

An inverse problem of reconstructing x such that $A(x,u)=0$ from the data y satisfying $C(u)=y$ can be written in the minimization form

$$\operatorname{argmin}_{(x,u)}\{\mathscr{J}(x,u;y):(x,u)\in M_{\mathrm{ad}}(y)\}.$$

The classical approaches usually consider $\mathscr{J}(x,u;y)=\frac{1}{2}\|C(x,u)-y\|^2$, $M_{\mathrm{ad}}=\{(x,u):A(x,u)=0\}$ or $\mathscr{J}(x,u;y)=\frac{1}{2}\|A(x,u)\|^2$, $M_{\mathrm{ad}}=\{(x,u):C(x,u)=y\}$. Here we follow a new approach assuming that the observation operator C can be inverted on its range, as is the case, e.g., in the practically relevant setting of a finite dimensional observation space. In this case, we can split u into two parts: the observed data part $\tilde{u}$ and the homogeneous data part $\hat{u}$, $u=\tilde{u}+\hat{u}$ where $C(\hat{u})=0$ and $C(\tilde{u})=y$. Let C^{ri} be an right inverse operator of C, then the problem becomes to find x such that

$$A(x,C^{\mathrm{ri}}(y)+\hat{u})=0,\quad \hat{u}\in\operatorname{Ker}(C):=\{\hat{u}:C(\hat{u})=0\}$$

with the new minimization form which is to find

$$(x, \hat{u}) \in \operatorname{argmin}_{(x,\hat{u})}\{\mathcal{J}(x, \hat{u}; y) : (x, \hat{u}) \in M_{\text{ad}}(y).$$

Under some more conditions on the smoothness of $\mathcal{J}$ with respect to y, we can prove the *well-definedness*, *stability* and *convergence* of minimizers.

In practice, often an iterative regularization method will be applied to reconstruct x. Here we want to mention the projected gradient method

$$x_{k+1} = \text{Proj}_{M_{\text{ad}}}(x_k - \mu_k \nabla J(x_k))$$

where $\text{Proj}_{M_{\text{ad}}}$ is the projection onto M_{ad} and x stands for $(x, \hat{u})$, for brevity. This method can be applied to many problems. Here we show numerical results with a Matlab implementation for three examples: inverse groundwater filtration (GWF), impedance acoustic tomography (IAT), and electrical impedance tomography (EIT).

We hope to see you there!

