

## INVITATION TO THE DOCTORAL SEMINAR

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"An extension of minimization based formulations and the projected gradient method with some applications"

## 9

https://classroom.aautit/Webkasday, 16 December 2020 3nh-a2g

**⊘** 10:00 a.m.

## Abstract

An inverse problem of reconstructing x such that A(x, u) = 0 from the data y satisfying C(u) = y can be written in the minimization form

 $\operatorname{argmin}_{(x,u)}\{\mathscr{J}(x,u;y):(x,u)\in M_{\operatorname{ad}}(y)\}.$ 

The classical approaches usually consider  $\mathscr{J}(x,u;y) = \frac{1}{2} ||C(x,u) - y||^2$ ,  $M_{ad} = \{(x,u) : A(x,u) = 0\}$  or  $\mathscr{J}(x,u;y) = \frac{1}{2} ||A(x,u)||^2$ ,  $M_{ad} = \{(x,u) : C(x,u) = y\}$ . Here we follow a new approach assuming that the observation operator *C* can be inverted on its range, as is the case, e.g., in the practically relevant setting of a finite dimensional observation space. In this case, we can split *u* into two parts: the observed data part  $\tilde{u}$  and the homogeneous data part  $\hat{u}$ ,  $u = \tilde{u} + \hat{u}$  where  $C(\hat{u}) = 0$  and  $C(\tilde{u}) = y$ . Let  $C^{ri}$  be an right inverse operator of *C*, then the problem becomes to find *x* such that

 $A(x, C^{ri}(y) + \hat{u}) = 0, \quad \hat{u} \in Ker(C) := \{\hat{u} : C(\hat{u}) = 0\}$ 

with the new minimization form which is to find

$$(x,\hat{u}) \in \operatorname{argmin}_{(x,\hat{u})} \{ \mathscr{J}(x,\hat{u};y) : (x,\hat{u}) \in M_{\operatorname{ad}}(y).$$

Under some more conditions on the smoothness of  $\mathscr{J}$  with respect to y, we can prove the *well-definedness*, *stability* and *convergence* of minimizers. In practice, often an iterative regularization method will be applied to reconstruct x. Here we want to mention the projected gradient method

$$x_{k+1} = \operatorname{Proj}_{M_{ad}}(x_k - \mu_k \nabla J(x_k))$$

where  $\operatorname{Proj}_{M_{ad}}$  is the projection onto  $M_{ad}$  and x stands for  $(x, \hat{u})$ , for brevity. This method can be applied to many problems. Here we show numerical results with a Matlab implementation for three examples: inverse groundwater filtration (GWF), impedance acoustic tomography (IAT), and electrical impedance tomography (EIT).

Barbara Kaltenbacher and the Department of Mathematics look forward to seeing you at the talk!

