Invitation to the Doctoral Seminar

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"An extension of minimization based formulations and the projected gradient method with some applications"

https://classroom.aau.at/b/kha-3nh-a2g

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Abstract

An inverse problem of reconstructing x such that $A(x,u) = 0$ from the data y satisfying $C(u) = y$ can be written in the minimization form

$$\operatorname{argmin}_{(x,u)}\{\mathscr{J}(x,u;y) : (x,u) \in M_{\text{ad}}(y)\}.$$

The classical approaches usually consider $\mathscr{J}(x,u;y) = \frac{1}{2}\|C(x,u) - y\|^2$, $M_{\text{ad}} = \{(x,u) : A(x,u) = 0\}$ or $\mathscr{J}(x,u;y) = \frac{1}{2}\|A(x,u)\|^2$, $M_{\text{ad}} = \{(x,u) : C(x,u) = y\}$. Here we follow a new approach assuming that the observation operator C can be inverted on its range, as is the case, e.g., in the practically relevant setting of a finite dimensional observation space. In this case, we can split u into two parts: the observed data part $\tilde{u}$ and the homogeneous data part $\hat{u}$, $u = \tilde{u} + \hat{u}$ where $C(\hat{u}) = 0$ and $C(\tilde{u}) = y$. Let C^{ri} be an right inverse operator of C, then the problem becomes to find x such that

$$A(x, C^{\text{ri}}(y) + \hat{u}) = 0, \quad \hat{u} \in \operatorname{Ker}(C) := \{\hat{u} : C(\hat{u}) = 0\}$$

with the new minimization form which is to find

$$(x,\hat{u}) \in \operatorname{argmin}_{(x,\hat{u})}\{\mathscr{J}(x,\hat{u};y) : (x,\hat{u}) \in M_{\mathrm{ad}}(y).$$

Under some more conditions on the smoothness of $\mathscr{J}$ with respect to y, we can prove the *well-definedness*, *stability* and *convergence* of minimizers.
In practice, often an iterative regularization method will be applied to reconstruct x. Here we want to mention the projected gradient method

$$x_{k+1} = \mathrm{Proj}_{M_{\mathrm{ad}}}(x_k - \mu_k \nabla J(x_k))$$

where $\mathrm{Proj}_{M_{\mathrm{ad}}}$ is the projection onto M_{ad} and x stands for $(x,\hat{u})$, for brevity. This method can be applied to many problems. Here we show numerical results with a Matlab implementation for three examples: inverse ground-water filtration (GWF), impedance acoustic tomography (IAT), and electrical impedance tomography (EIT).

Barbara Kaltenbacher and the Department of Mathematics look forward to seeing you at the talk!