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“On the Stable Set and Graph Coloring Problems: Semidefinite Relaxations and a Quantum Annealing Approach”

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Abstract

The stable set problem and the graph coloring problem are two fundamental problems in combinatorial optimization. The stable set problem aims to determine the size of the largest set of non-adjacent vertices, called the stability number of the graph, while the graph coloring problem seeks the minimum number of colors required to color the vertices of a graph such that no two adjacent vertices share the same color, referred to as the chromatic number of the graph. Motivated by the practical importance and computational hardness of these problems, we investigate both classical and emerging methods for solving these problems, focusing on semidefinite relaxations and quantum annealing. The Lovász theta function is a fundamental semidefinite programming relaxation that provides bounds on both the stability number and the chromatic number of a graph. We propose to strengthen these bounds by introducing new valid inequalities into the semidefinite program for its computation. For this purpose, we focus on subgraphs containing cliques, odd cycles, and odd antiholes. Our computational experiments demonstrate that these enhancements significantly improve the bounds for both problems while maintaining computational efficiency. Furthermore, we investigate the exact subgraph hierarchy, a semidefinite programming hierarchy based on the Lovász theta function that provides increasingly tighter upper bounds on the stability number of graphs. We examine its behavior for Paley graphs, a class of strongly regular, vertex-transitive graphs, and show that the bounds on their stability number do not improve upon the Lovász theta function past a certain threshold. To address this limitation, we introduce a new variant of the exact subgraph hierarchy tailored to vertex-transitive graphs. Our theoretical and computational results confirm that this hierarchy yields competitive bounds for vertex-transitive graphs. Finally, we explore quantum annealing for solving the stable set problem using the DWave quantum annealer. We formulate the problem as a quadratic unconstrained binary optimization problem with the penalty method and investigate how to set the penalty term to match the graph's stability number. Given the heuristic nature of the D-Wave system, we develop methods for enhancing solutions and strategies for handling larger instances. A comprehensive computational analysis shows that the proposed methods improve solution quality and enable the approach to be extended to larger instances. Altogether, through strengthened semidefinite relaxations, the introduction of a new variant of the exact subgraph hierarchy for vertex-transitive graphs, and the investigation of quantum annealing techniques, this thesis advances both classical and emerging approaches for tackling hard combinatorial optimization problems.

Angelika Wiegele and the Department of Mathematics look forward to seeing you at the talk!