Invitation to the Defense

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"Imaging with Nonlinear Ultrasound Waves: Modeling, Analysis and Numerics"

HS 5 | Tuesday, 7 October 2025 | 3:00 p.m.

Abstract

The research presented in this thesis is motivated by a range of biomedical applications in which nonlinear acoustic wave phenomena play a central role. In particular, the focus is on two advanced ultrasound imaging techniques: ultrasound vibro-acoustography and contrast-enhanced ultrasound, which involve inverse and forward problems governed by nonlinear partial differential equations and coupled systems. The first part investigates paraxial models for ultrasound vibro-acoustography. Beginning with a general second-order nonlinear wave equation, we introduce a perturbative framework and apply a paraxial transformation to derive directed beam models. To enable efficient analysis and reconstruction, we transition to the frequency domain and focus on a representative model. The associated inverse problem is the reconstruction of the nonlinearity parameter $B/A(x)$, for which we develop a Landweber–Kaczmarz iteration scheme. We further study a nonlinear inverse problem involving the simultaneous recovery of the sound speed $c(x)$ and the nonlinearity parameter $B/A(x)$. The forward model is adapted to account for spatial variability in the sound speed. We establish the differentiability of the forward operator and prove convergence

of a Newton-type method for solving the inverse problem in an all-at-once formulation. In the second part we consider contrast-enhanced ultrasound, where gas-filled microbubbles serve as nonlinear contrast agents. These interactions are modeled using coupled PDE-ODE systems. We first derive a hierarchy of such models and then focus on a system coupling the Westervelt equation for acoustic pressure with a Rayleigh–Plesset type equation for bubble dynamics in terms of the radius. For this coupled model, we rigorously establish the local-in-time existence of solutions. Numerical experiments further illustrate both single-bubble dynamics and their influence on wave propagation. To address computational challenges of these coupled systems in the time domain, we introduce a multiharmonic approximation framework under time-periodic excitation. In this setting, the Westervelt equation is coupled to a volume-based ODE approximation. We establish the existence of time-periodic solutions and develop efficient numerical schemes using truncated Fourier representations. Simulations show how microbubble effects and the number of harmonics influence nonlinear wave behavior.

Roswitha Rissner and the Department of Mathematics look forward to seeing you at the talk!