ALPEN-ADRIA UNIVERSITÄT KLAGENFURT

Invitation to the Defense

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"On the Asymptotic Structure of Powers of Monomial Ideals"

N.1.42 | Monday, 6 October 2025 | 10:00 a.m.

Abstract

This thesis investigates the asymptotic behaviour of algebraic invariants associated with powers of monomial ideals, with particular emphasis on associated primes and the number of minimal generators.

After introducing the necessary background on monomial ideals and their combinatorial interpretations, we investigate the number of generators of high powers of bivariate monomial ideals. We establish an explicit bound beyond which the number of minimal generators becomes polynomial in the power, and provide a method for constructing the minimal generating sets of these powers from certain subideals of a fixed power, thereby reducing computational complexity. These results facilitate the effective computation of Hilbert functions and related invariants.

We then study the structure of Buchberger graphs and their relation to associated primes, introducing the lcm-complex to generalize results from strongly generic to arbitrary monomial ideals. This framework allows for a combinatorial characterization of associated primes in terms of simplicial complexes derived from least common multiples of the minimal generators of an ideal.

For monomial ideals in three variables, we apply the structural results from the bivariate case to derive bounds on the stability index of associated primes, considering cases based on the number and structure of minimal primes. Finally, we turn to monomial ideals in an arbitrary number of variables and address the problem of bounding the copersistence index—the power after which the sequence of associated primes of powers of an ideal is weakly decreasing. We present a method to derive bounds based on systems of linear inequalities that encode information about associated primes. Our approach yields upper bounds for the copersistence index that improve the existing bound by an exponential factor.
These results contribute to a better understanding of the asymptotic properties of monomial ideals, particularly the behaviour of their associated primes and minimal generators under powers, and provide new tools for their analysis within both algebraic and combinatorial frameworks.

Roswitha Rissner and the Department of Mathematics look forward to seeing you at the talk!