ALPEN-ADRIA UNIVERSITÄT KLAGENFURT

Invitation to the Defense

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"Iterative decomposition methods for ill-posed problems"

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Abstract

The aim of this work is to develop and analyze iterative regularization methods, which additionally decompose the approximate solutions of ill-posed problems based on specific criteria. More precisely, we consider two different ways of decomposition. First, hierarchical decomposition means expressing reconstructed solutions as sums of components, where each component contains information at a different level of detail. Secondly, structural decompositions aim for components that represent different types of features, such as smooth, piecewise constant or oscillatory parts. We start by investigating the multiscale hierarchical decomposition method (MHDM), which was originally introduced for total variation image denoising and deblurring. Using a monotone sequence of regularization parameters, the method iteratively computes solutions of Tikhonov-type variational problems with the purpose of adding details that would be considered noise at a different scale. We consider the method for a large class of not necessarily convex penalties, and extend convergence results of the residual, as well as a multiscale representation of the data to our setting. We then compare the method to single-step Tikhonov regularization and propose necessary and sufficient conditions when the iterates of the MHDM correspond to the minimizers of a Tikhonov problem with a suitable regularization parameter. As a next step, we adapt the MHDM to the highly ill-posed, nonlinear problem of blind image deblurring. This means, given a blurry and noise-corrupted image, we aim to find a kernel function and an approximation of the true image such that their convolution matches the observation. The novelty of our approach is decomposing both the approximate image and the blurring kernel additively, such that more details are added consecutively. We consider Sobolev norm regularizers and show that the iterates can be chosen such that the Fourier transform of the kernel is real-valued and non-negative. Moreover, this result enables us to compute the iterates pointwise in the Fourier domain. For decomposing solutions in a structural, instead of a hierarchical, manner, we introduce nested Bregman iterations. The idea is to combine Bregman iterations with Morozov regularization for an infimal convolution of penalties. This means we iteratively solve constrained minimization problems where one of the infimally convolved regularizers is replaced by a suitable Bregman distance. Thus, at each iteration, we obtain a regularized solution consisting of two components, where the contribution of each individual component to the overall solution is varied within the iterations. Effectively, this transforms the problem of choosing weighting parameters for the infimal convolution into stopping the iteration appropriately. We discuss the well-definedness and the convergence behavior of the proposed method and propose a cross-correlation stopping criterion. We conduct extensive numerical experiments for all proposed procedures to illustrate their behavior and robustness. In particular, the iterative nature of the methods allows to reconstruct reasonable solutions by simply terminating appropriately, instead of tuning regularization parameters.

Viktoriia Grushkovska and the Department of Mathematics look forward to seeing you at the talk!