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Invitation to the Defense

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"From Linear Orderings to Infinite Trees: Semidefinite Programming for Combinatorial Optimization and Extremal Combinatorics"

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Abstract

A combinatorial optimization problem aims to optimize a function over a discrete set, subject to some constraints. Many problems in daily life can be formulated as such, making them an important topic in the field of optimization. However, they are usually very hard to solve. We aim to formulate relaxations of these problems, which are easier to solve and provide bounds approximating the optimal solutions. One powerful tool to obtain good relaxations of combinatorial optimization problems is semidefinite programming. In this thesis, we use various methods to derive and improve semidefinite programming relaxations of four types of problems in combinatorial optimization and extremal combinatorics.

The first problem we consider is the Minimum Cutwidth Problem. It is a well-known NP-hard problem, aiming to find the linear ordering (or permutation of vertices) of a graph that minimizes a linear function called the cutwidth. To obtain lower bounds for this problem, we introduce a semidefinite relaxation, which we then strengthen with several classes of valid inequalities. We also construct upper bounds from the solution of the

semidefinite program. Computational experiments show that, for dense graphs, our algorithm outperforms state-of-the-art solvers.

The second problem lies in extremal combinatorics. We consider leaf-labeled rooted binary trees, a type of tree in which only leaves are vertices. We can define notions of density for these trees and study how these densities behave in trees whose sizes grow to infinity. We adapt Razborov's flag algebra theory to this setting, allowing us to use polynomial optimization methods to formulate semidefinite relaxations of these problems. This enables us to obtain new results on the densities of trees in a systematic way.

Next, we consider three variants of the famous Ramsey numbers: the ordered and unordered canonical Ramsey numbers, and the ordered Ramsey numbers. We use several flag algebra variations to formulate semidefinite programs that provide upper bounds for each of these numbers. We also obtain lower bounds on these numbers by implementing integer linear programming formulations and heuristics. With these methods, we can determine the exact values of these Ramsey numbers variants for many small graphs.

Finally, we turn our attention to the set of products of box-constrained vectors of dimension n, frequently encountered in relaxations of combinatorial optimization problems. To better approximate the convex hull QPBn of this set, we introduce new valid linear inequalities obtained by studying the polyhedral structure of a known disjunctive characterization for QPB3. We also derive new second-order cone constraints of QPBn from these linear inequalities. We show computationally that the new inequalities yield exact solutions for instances that were not solved exactly with previously known constraints.

Andrei Asinowski and the Department of Mathematics look forward to seeing you at the talk!