ALPEN-ADRIA UNIVERSITÄT KLAGENFURT

INVITATION TO THE DEFENSE

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"A Provably Good Connection: Spanning Trees, Edge Expansion, and Semidefinite Programming"

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Abstract

The edge expansion problem and the quadratic minimum spanning tree problem (QMSTP) are two well-known, hard combinatorial problems in graph theory. In the edge expansion problem, one wants to find a cut in the graph, minimizing the ratio between the size of the cut and the number of vertices on the smaller side of the cut. The quadratic minimum spanning tree problem is to minimize a quadratic cost function over all possible spanning trees of a graph. To solve these two problems, algebraic connectivity and semidefinite programming are essential tools.

We present two variants of exact algorithms using semidefinite programming (SDP) to compute the edge expansion of a graph. The first variant uses the SDP relaxation first to reduce the search space considerably. We give two different SDP-based versions to then compute the edge expansion. Our second variant to compute the edge expansion uses Dinkelbach's algorithm for fractional programming. Here we again use an SDP-based state-of-the-art solver to obtain solutions of the parametrized subproblems. Numerical results demonstrate that with our algorithms, one can compute the edge expansion on graphs with up to 400 vertices in a routine way, including

instances where standard branch-and-cut solvers fail. To the best of our knowledge, these are the first SDP-based solvers for computing the edge expansion of a graph.

A different direction of tackling the problem is to apply a recently developed convexification technique for fractional programs by He, Liu, and Tawarmalani (2024) to the edge expansion problem. We give a formulation of the edge expansion as a completely positive program and propose a relaxation as a doubly non-negative program, further strengthened by cutting planes. We develop an alternating direction method of multipliers (ADMM) and an augmented Lagrangian algorithm to solve the doubly non-negative program, obtaining lower bounds on the edge expansion. Numerical results confirm that this relaxation yields strong bounds and the augmented Lagrangian algorithm is computationally efficient.

For the QMSTP, we derive three exact formulations as mixed-integer SDPs exploiting the algebraic connectivity of a graph and the line graph of a connected graph. Based on these formulations, we derive a doubly non-negative relaxation for the QMSTP and investigate classes of valid inequalities to strengthen the relaxation using the Chvátal–Gomory procedure for mixed-integer conic programming. We develop and implement a version of the Peaceman–Rachford splitting method that allows us to compute the new bounds for graphs from the literature in reasonable time. The numerical results demonstrate that our bounds significantly improve over existing bounds from the literature in both quality and computation time, in particular for graphs with more than 30 vertices.

This thesis provides further evidence to the growing pool of problems for which semidefinite programming is used as a valuable tool for combinatorial optimization.

Barbara Kaltenbacher and the Department of Mathematics look forward to seeing you at the talk!