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INVITATION TO THE DOCTORAL SEMINAR

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"The randomized quadrature and randomized numerical schemes"

N.1.44 | Wednesday, 15 January 2025 | 10:00 a.m.

Abstract

In this talk, I will explain how stratified Monte Carlo simulation can be used to develop randomized quadrature methods for time integration (1D case) and integration over a 2D domain (2D case). Building on these foundations, I will present randomized numerical schemes applicable to a broad class of deterministic differential equations with irregular coefficients. The initial schemes considered are randomized Euler and randomized Runge-Kunta for Caratheodory ODEs. Our motivation for studying Caratheodory type initial value problems stems from the fact that certain rough differential equations that are driven by an additive noise can be transformed into a problem of this form. It is well-known that there is lack of convergence for deterministic algorithms in this case, and our proposed methods in [1] showed that even with very mild conditions, the order of convergence can be at least half with respect to the Lp norm. We then investigated the applications of two randomized quadrature formulas (2D case) to the finite element method (FEM) for elliptic boundary value problems with irregular coefficient functions in [2]. In general, the entries of this matrix-vector system of FEM are not known explicitly but need to be approximated by

quadrature rules. If the coefficient functions of the differential operator or the forcing term are irregular, then standard quadrature formulas, such as the barycentric quadrature rule, may not be reliable. Our analysis showed that the convergence rate of our proposed quadrature can be at least half-2/p even if the force term is only Lp integrable. Recently, we pushed this idea in [1] to Caratheodory delay ODEs (CDDEs), where a randomised Euler scheme is proposed to approximate the exact solution [3]. It is worth mentioning that, mainly due to CDDEs being considered interval-by-interval, we developed a suitable proof technique that is based on mathematical induction. The randomised technique from [1] is only applicable for the initial inductive step; as the systems iterate over time, a different strategy is required to handle the effect of the delay variable.

[1] Kruse, R. and Wu, Y., 2017. Error analysis of randomized Runge–Kutta methods for differential equations with time-irregular coefficients. Computational Methods in Applied Mathematics, 17(3), pp.479-498. [2] Kruse, R., Polydorides, N. and Wu, Y., 2019. Application of Randomized Quadrature Formulas to the Finite Element Method for Elliptic Equations. arXiv preprint arXiv:1908.08901. [3] Difonzo, F.V., Przybyłowicz, P. and Wu, Y., 2024. Existence, uniqueness and approximation of solutions to Carathéodory delay differential equations. Journal of Computational and Applied Mathematics, 436, 115411.

Michaela Hitz and the Department of Statistics look forward to seeing you at the talk!