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Invitation to the Colloquium

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"On the algebraic and arithmetic structure of commutative rings"

HS 2 Monday, 19 February 2024 2:30 p.m.

Abstract

Commutative rings are one of the main research objects of modern algebra. Historically, their systematic study began with Richard Dedekind's introduction of so-called *ideals*. At the time, mathematicians discovered that the fundamental theorem of arithmetic—unique factorization of natural numbers into prime powers—is in general not transferable to other number domains. In particular not to those which occurred in the quest of proving Fermat's last theorem.

The existence of (unique) factorization of elements into products of indecomposable components, or lack thereof, is one of the characteristics of the arithmetic structure of a ring. While Dedekind's motivation was of number-theoretic nature, his axiomatic approach and his level of abstraction inspired many mathematicians. The concepts he introduced turned out to be groundbreaking. By the 20th century, ideal theory had been used in a wide variety of problems.

For above-mentioned number domains, Dedekind showed that the ideal-theoretic version of the fundamental theorem of arithmetic holds: ideals uniquely factor into *prime ideals*, which are the analog of prime numbers in

the setting of ideals.

However, there occur other commutative rings in mathematics which are not, what is today called, *Dedekind domains*. In the late 20th century, the study of elementwise factorization in commutative rings received more and more attention. Although these studies appear to be purely algebraic and number-theoretic at first sight, other mathematical disciplines, such as combinatorics, graph theory, and symbolic computation, play a recurring role.

In this line of research, one major goal is to understand all the factorizations of one element into its indecomposable components—the irreducible elements. A family of rings which are known for their fascinating factorization behavior are the rings $\mathrm{Int}(D)$ of integer-valued polynomials over a domain D. Due to the factorization-theoretic complexity, their arithmetic structure is not yet fully understood. Part of the speaker's research addresses the construction of elements with prescribed factorization behavior in a wide variety of these rings. These results once more substantiate the perception that there are no limits to the non-uniqueness of factorizations in rings of integer-valued polynomials.

A phenomenon in this context is the following: It is possible that an irreducible element c has a power c^k which can be decomposed in a different way than the obvious one, that is, $c \cdot c \cdots c$. The ones for which all powers factor uniquely are called *absolutely irreducible*. Identifying these elements is in general hard but crucial for a complete picture of the arithmetic structure. The speaker's research significantly contributes to the field, among the highlights are the verification of the long-open conjecture that the binomial polynomials are absolutely irreducible in $\mathrm{Int}(\mathbb{Z})$ and the complete characterization of the absolutely irreducibles in $\mathrm{Int}(D)$ where D is a discrete valuation domain.

Angelika Wiegele and the Department of Mathematics look forward to seeing you at the talk!