ALPEN-ADRIA UNIVERSITÄT KLAGENFURT

Invitation to the Doctoral Seminar

Łukasz Stępień, MS.c.

AGH University of Science and Technology in Kraków

"On global approximation of SDEs driven by countably dimensional Wiener process"

HS 7 Wednesday, 18 October 2023 11:00 a.m.

Abstract

In this talk, we consider global approximation of solutions of the following stochastic differential equations (SDEs)

$$\begin{cases} \mathrm{d}X(t) = a(t, X(t))\mathrm{d}t + \sigma(t)\mathrm{d}W(t), \ t \in [0, T], \\ X(0) = x_0, \end{cases}$$

where $T > 0$, $W(t) = [W_1(t), W_2(t), \ldots]^T$ is a sequence of independent scalar Wiener processes on the probability space with sufficiently rich filtration, and $x_0 \in \mathbb{R}$. For suitably regular coefficients a, σ, the uniqueness of the solution $X = X(t)$ and its finite second-order moments can be assured [2, 6]. The models with countable Wiener process allow to describe sophisticated random noise structure and hence, can be used in e.g., financial modelling and genetics [1, 3].

We focus on recent results announced in [7] where we show lower bounds for exact asymptotic error behaviour in the spirit of Information-Based Complexity (IBC) framework [8]. For that reason, we analyse separately two classes of admissible algorithms: leveraging discrete equidistant infor-

mation about the truncated Wiener process, and those possibly based on adaptive/non-equidistant meshes. Our results indicate that in both cases, decrease of any method's error requires significant increase of the cost term, which is illustrated by the product of cost and error diverging to infinity. This is, however, not visible in finite dimensional case [4, 5]. We also construct an implementable, path-independent Euler algorithm with adaptive step-size control, which is asymptotically optimal among algorithms using specified truncation levels of the underlying Wiener process. Ultimately, we present results of numerical simulations performed in the Python language.

References

[1] Carmona, R., Teranchi, M.: *Interest Rate Models: An Infinite Dimensional Stochastic Analysis Perspective*, Springer, Berlin (2006).

[2] Cohen, S.N., Elliot, R.J.: *Stochastic calculus and applications, 2nd ed.*, Probability and its applications, Birkhäuser, New York, (2015).

[3] Da Prato, G., Zabczyk, J.: *Stochastic Equations in Infinite Dimensions, second ed.*, Cambridge University Press, 2014.

[4] Hofmann, N., Müller–Gronbach, T., Ritter, K.: Optimal approximation of stochastic differential equations by adaptive step-size control, *Math. Comput.*, **69** (1999), 1017–1034.

[5] Kałuża, A., Przybyłowicz, P.: Optimal global approximation of jump-diffusion SDEs via path-independent step-size control, *Appl. Numer. Math.* **128** (2018), 24–42.

[6] Przybyłowicz, P., Sobieraj, M., Stępień, Ł.: Efficient approximation of SDEs driven by countably dimensional Wiener process and Poisson random measure, *SIAM J. Numer. Anal.* **60** (2022), 824-855.

[7] Stępień, Ł., Adaptive step-size control for global approximation of SDEs driven by countably dimensional Wiener process, preprint, March 2023, arXiv:2303.13171

[8] Traub, J.F., Wasilkowski, G.W., Woźniakowski, H.: *Information-Based Complexity*, Academic Press, New York (1988).

Michaela Szölgyenyi and the Department of Statistics look forward to seeing you at the talk!