1. The pages of a notebook are numbered consecutively such that the first sheet contains the numbers 1 and 2, the second sheet contains the numbers 3 and 4, and so on. One sheet is torn out of the notebook. The page numbers on the remaining sheets are added. The resulting sum equals 2021.

   (a) How many pages can the notebook have had originally?

   (b) Which page numbers could be found on the sheet that has been torn out?

   (Walther Janous)

2. Let ABC be a triangle with circumcenter U such that $\angle CBA = 60°$ and $\angle CBU = 45°$. Let D be the point of intersection of the lines BU and AC.
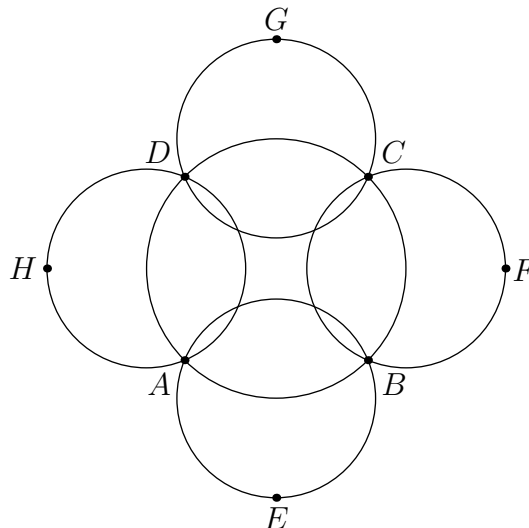
   Prove that $AD = DU$.

   (Karl Czakler)

3. The eight points A, B, ..., G and H are placed on five circles as in the figure below. Each of these letters will be replaced with one of the numbers 1, 2, ..., 7 and 8 such that the following two conditions hold:

   (i) Each of the eight numbers is used exactly once.

   (ii) The sum of the numbers on each of the five circles is the same.

   How many possibilities are there to replace the letters with numbers in this way?

   (Walther Janous)

4. Let p be a prime and let m and n be positive integers such that $p^2 + m^2 = n^2$.

   Prove that $m > p$.

   (Karl Czakler)

Working time: 4 hours.
Each problem is worth 8 points.