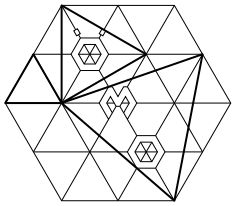

50th Austrian Mathematical Olympiad

Junior Regional Competition

18th June 2019

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1. Determine all pairs (x, y) of integers with $x + y \neq 0$ which are satisfying

$$\frac{x^2 + y^2}{x + y} = 10.$$

(Walther Janous)

2. Let $ABCD$ be a square. The equilateral triangle BCS is constructed on the exterior of the side BC. Let N denote the midpoint of the line segment AS and let H be the midpoint of the side CD.

   Prove: $\angle NHC = 60°$.

(Karl Czakler)

3. Alice and Bob play a game that allows the playing numbers 19 and 20 and the two possible starting numbers 9 and 10. Alice chooses her playing number and assigns the remaining playing number to Bob while Bob independently chooses the starting number.

   Alice adds her playing number to the starting number, Bob adds his playing number to the sum, then Alice again adds her playing number to this new sum and so on. The game lasts till the number 2019 is reached or exceeded.

   A player who obtains exactly 2019 wins. If 2019 is exceeded, the game ends in a draw.

   - Show that Bob cannot win.
   - Which starting number does Bob have to choose in order to prevent Alice from winning?

(Richard Henner)

4. Let p, q, r and s be prime numbers satisfying

$$5 < p < q < r < s < p + 10.$$

   Prove that the sum of these four prime numbers is divisible by 60.

(Walther Janous)

Working time: 4 hours.
Each problem is worth 8 points.