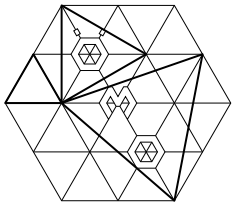

57th Austrian Mathematical Olympiad

Regional Competition

26th March 2026

1. Let a and b be positive real numbers with

$$(a+4)(b+1) = 24.$$

Prove the inequality

$$a^2 + b^2 \geq 13.$$

When does equality hold?

(Karl Czakler)

2. Let k be the circumcircle of a square $ABCD$. Let P be a point on the shorter arc CD of k with $P \neq C$ and $P \neq D$. Let the intersection of the line BP with the line AC be Q, and let the intersection of the line CP with the line AD be R.

Prove that the line RQ is perpendicular to the line AC .

(Karl Czakler)

3. Let n be a positive integer. A set of positive integers is called *airy*, if it does not contain two consecutive numbers. For each airy subset of the set $\{1, 2, \ldots, n\}$ compute the product of its elements, then add the squares of all these products where the empty subset is assigned the product 1.

Determine the sum s_n.

Example: $n = 3$

The airy subsets of $\{1, 2, 3\}$ are $\{\}$, $\{1\}$, $\{2\}$, $\{3\}$ und $\{1, 3\}$. For the products, we get

$\{\}$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,3\}$
1	1	2	3	3

Therefore, the sum of the squares equals $s_3 = 1^2 + 1^2 + 2^2 + 3^2 + 3^2 = 24$.

(Walther Janous)

4. Prove the following two assertions.

   (a) There are infinitely many square numbers of the form $3^k + 3^n$ with positive integers k and n.

   (b) There are no square numbers of the form $7^k + 7^n$ with positive integers k and n.

(Walther Janous)

Working time: 4 hours.
Each problem is worth 8 points.