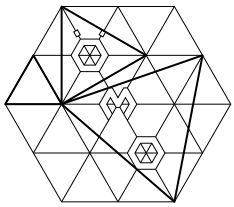

50th Austrian Mathematical Olympiad

Regional Competition

4th April 2019

---

1. Let x and y be real numbers satisfying $(x+1)(y+2)=8$.

   Show that

   $$(xy-10)^2 \geq 64.$$

   Furthermore, determine all pairs (x,y) of real numbers for which equality holds.

   (Karl Czakler)

2. Let $ABCDE$ be a convex pentagon having a circumcircle and satisfying $AB=BD$. The point P is the intersection of the diagonals AC and BE. The lines BC and DE intersect in point Q.

   Show that the line PQ is parallel to the diagonal AD.

   (Gottfried Perz)

3. Let $n \geq 2$ be an integer.

   We draw an $n \times n$ grid on a board and label each box with either the number -1 or the number 1. Then we calculate the sum of each of the n rows and the sum of each of the n columns and determine the sum S of these $2n$ sums.

   (a) Show that there does not exist a labelling of the grid with $S=0$ if n is odd.

   (b) Show that there exist at least six different labellings with $S=0$ if n is even.

   (Walther Janous)

4. Determine all non-negative integers n smaller than 128^{97} which have exactly 2019 positive divisors.

   (Richard Henner)

Working time: 4 hours.
Each problem is worth 8 points.