

57th Austrian Mathematical Olympiad

National Competition—Final Round (Day 1)

27th May 2026

1. Let ABC be an acute triangle with orthocenter H and circumcircle k . Let S be the intersection point of the tangents to k at points A and B , let M be the midpoint of AB , and let H' be the reflection of H with respect to AB . Let P be the intersection point of the line MH with the circumcircle k such that H lies between M and P .

Prove that the points S , H' and P are collinear.

(Karl Czakler)

2. Alice and Bob play a game. In the beginning, there is a pile of n stones on the table, where n is a positive integer. In each move, either one stone is removed from an arbitrary pile, or one pile is split into two non-empty piles. The two players alternate, and Alice begins. When a player has no valid moves, the other player wins.

Determine for each n which of the two players has a winning strategy.

(Theresia Eisenkölbl)

3. Determine all values a and b such that there exists a sequence $(u_n)_{n \geq 1}$ of positive integers with $u_1 = a$, $u_2 = b$, and

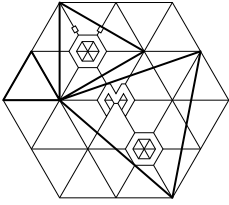
$$u_{2n} = \text{GM}(u_{2n+1}, u_{2n-1}) \quad \text{and} \quad u_{2n+1} = \text{AM}(u_{2n}, u_{2n+2}) \quad \text{for } n \geq 1.$$

Here, $\text{AM}(x, y)$ denotes the arithmetic and $\text{GM}(x, y)$ the geometric mean of the two numbers x and y .

(Theresia Eisenkölbl)

Working time: $4\frac{1}{2}$ hours.

Each problem is worth 8 points.



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4. Determine all real numbers α for which a function $f: \mathbb{R} \rightarrow \mathbb{R}$ exists such that

$$f(f(x) + y) = \alpha x + f(x + f(f(y)))$$

for all real numbers x and y .

(Walther Janous)

5. Let $ABCDEF$ be a convex hexagon with the property that each of the three segments joining the midpoints of opposite sides divides the total area of the hexagon in half.

Show that the three segments intersect in a common point.

(Walther Janous)

6. On two strips, each consisting of 100 cells numbered from 1 to 100, there are 50 tokens each.

On the first strip, the tokens are located on cells 1, 3, 5, \dots , 99. Each token is then moved one or more cells to the right, i. e., to a cell with a higher number. The tokens may not jump over each other, and no cell may contain two or more tokens. Let A be the number of possible configurations of the tokens that can be reached this way.

On the second strip, the tokens are located on cells 1, 2, 3, \dots , 50. Again, each token is moved one or more cells to the right. The tokens may not jump over each other, and no cell may contain two or more tokens. Moreover, a token cannot be moved further than to the position double its starting position. Let B be the number of possible configurations for the tokens that can be reached this way.

Prove that $A = B$ holds.

(Stephan Wagner)

Working time: $4\frac{1}{2}$ hours.

Each problem is worth 8 points.