57^{th} Austrian Mathematical Olympiad

Junior Regional Competition

16th June 2026

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1. Let a, b, c be positive real numbers with $a + b \neq c$, $b + c \neq a$ and $c + a \neq b$.

   Prove that at least one of the three numbers

   $$\frac{c}{a+b-c}, \quad \frac{a}{b+c-a}, \quad \frac{b}{c+a-b}$$

   is less than or equal to 1.

   (Karl Czakler)

2. Let $ABCD$ be a trapezoid with $AB \parallel CD$, $AB > CD$ and $\angle BAD + \angle CBA = 90°$. Furthermore, let M and N be the midpoints of the sides AB and CD, respectively.

   Prove:

   $$2 \cdot MN = AB - CD.$$

   (Karl Czakler)

3. Every tile in a game of dominoes is made of two halves. Each half displays one of the numbers 0, 1, …, 6. Every tile $\boxed{a|b}$ with $0 \leq a \leq b \leq 6$ occurs exactly once. We consider $\boxed{b|a}$ to be the same tile as $\boxed{a|b}$.

   A *row* is formed by placing at least two tiles after each other in such a way that the numbers on neighboring halves of consecutive tiles are always equal.

   A row is called *closed* if the numbers on both ends of the row are also equal.

   For example: $\boxed{0|2}$, $\boxed{2|4}$, $\boxed{4|4}$, $\boxed{4|0}$ is a closed row.

   Determine the maximum number of closed rows that can be formed simultaneously with this game of dominoes. (It is allowed to have some tiles left over.)

   (Karl Czakler)

4. Determine all positive integers k such that

   $$2^k + 3^k + 4^k$$

   is a perfect square.

   (Walther Janous)

Working time: 4 hours.
Each problem is worth 8 points.