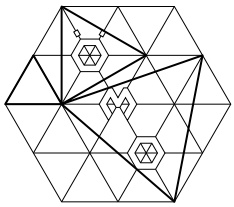

57th Austrian Mathematical Olympiad

National Competition—Preliminary Round

2nd May 2026

1. Prove that for all integers $n \geq 2$ the inequality

$$\sqrt[2]{2+\sqrt[3]{3+\sqrt[4]{\ldots+\sqrt[n]{n}}}} < 2$$

holds.

(Walther Janous)

2. Let ABC be an acute triangle. Let D be a point in its interior such that

$$\angle DBA + 2 \cdot \angle BAC + \angle ACD = 180^\circ.$$

Let I, J and K denote the midpoints of segments DA, DB and DC, respectively.

Show that there exists a point X, independent of the choice of D, which lies on the circumcircle of triangle IJK.

(Dominik Pultar)

3. A list of numbers is called *productive* if it includes two distinct numbers whose product is included in the list.

What is the smallest number k such that we can obtain a list that is not productive by removing k numbers from the list

$$1, 2, \ldots, 2025, 2026\,?$$

(Walther Janous)

4. A sequence $(a_n)_{n\geq 0}$ of positive integers is called *k-beautiful* if, for all $n \geq 0$, the property $a_n = a_{n+k}$ and, for all $n \geq 1$, the property

$$a_{n-1} + a_{n+1} \mid a_n^2 + k$$

is satisfied. For which positive integers k is there a k-beautiful sequence?

(Dominik Pultar, Jan Strehn)

Working time: $4\frac{1}{2}$ hours.
Each problem is worth 8 points.