



56th Austrian Mathematical Olympiad

Junior Regional Competition

12th June 2025

1. Determine all triples (a, b, c) where $a, b, c \geq 0$ are integers which satisfy

$$ab + bc + ca = 2(a + b + c).$$

(Karl Czakler)

2. Let ABC be an acute-angled triangle. Let the points D, E and F be the feet of the altitudes through the vertices A, B and C .

Show: If $AF = CF$, then the segments DE and DF are perpendicular.

(Karl Czakler)

3. How many four-digit-numbers are there in the decimal system such that the thousands digit is greater than all other digits?

(Gerhard Kirchner)

4. Consider all numbers of the form

$$20^n + 2 \cdot 5^n,$$

where $n \geq 2025$ is an integer.

Determine the largest integer k that divides each of these numbers.

(Walther Janous)

Working time: 4 hours.

Each problem is worth 8 points.